3-D Digital Modeling and Giuliano da Sangallo's Designs for Santa Maria delle Carceri in Prato

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Giuliano da Sangallo (ca. 1445–1516) was an architect capable of managing the entire architectural process, from conception through realization. The pilgrimage church of Santa Maria delle Carceri in Prato (begun 1485), a milestone in Italian Renaissance debates around Greek cross-shaped churches, reveals both Sangallo's competence in building complex structural elements and his adherence to Leon Battista Alberti's theories (Figure 1). The complexity of Sangallo's design process makes it difficult to understand many of the architectural issues he tackled in this project, the solutions proposed in his drawings, and the eventual translation of his drawings into brick and stone. In this article, we overcome these difficulties by using three-dimensional models to analyze the church and its design process (Figure 2). Typically, a digital 3-D model of a building results from the virtual assembly of many architectural components. When those individual elements are coded to identify their mutual structural relationships (for example, where columns bear the load of a lintel, or where a cupola distributes its weight onto a drum), the 3-D model can be described as a semantic data model.[1]

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Figure 1 Giuliano da Sangallo, Santa Maria delle Carceri, Prato, begun 1485, current front elevation (authors' photo).

Our decision to use 3-D modeling to examine Santa Maria delle Carceri was based not only on the increasing popularity and reliability of these digital tools but also on the fact that such modeling bears important similarities to the methods used by Sangallo himself.[2] Trained as a woodcutter, or *legnaiolo*, he was a skilled model maker. Three-dimensional models afforded him a powerful way to explain projects to customers and supply accurate instructions to craftsmen; he also used them to research new technical and formal solutions, making sketches to conceive 3-D shapes.[3] An excellent

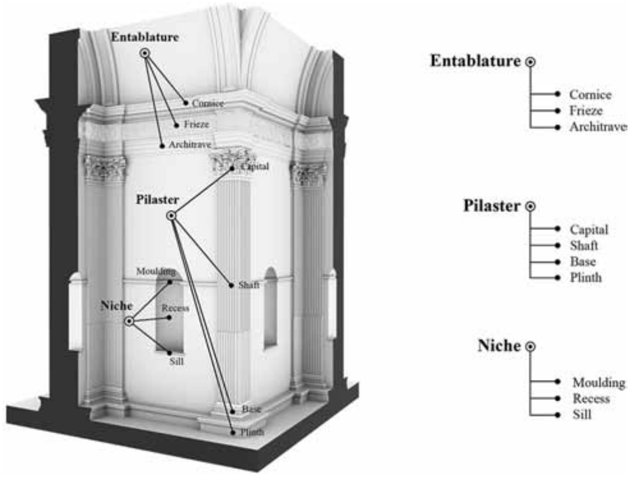

Figure 2 Digital semantic 3-D model of an area of the sanctuary of Santa Maria delle Carceri, Prato, with its 3-D graph (image by the authors).

Figure 3 Giuliano da Sangallo, Santa Maria delle Carceri, Prato, begun 1485, current right-side elevation (authors' photo).

illustration of Sangallo's process is provided by the case of Santa Maria delle Carceri, where a series of 3-D models created by the architect and mentioned in the archives reflect a number of changes requested by the patrons, the *operai* of the church.

For researchers attempting to answer specific reconstruction questions using incomplete data, today's digital models offer distinct advantages over the sorts of concrete physical models that Sangallo employed. For one thing, digital 3-D models bypass problems of representational scale, as they are built virtually at 1:1 scale. Conventional scale models allow for the study of problems and solutions, but the solutions often are not translatable to larger scales. 3-D models also offer a way to work around the incompleteness of drawings: because of their nonscalar properties, they allow modern scholars to reconstruct historic buildings using the same rules the architects did (in this case, as described by Giuliano da Sangallo in his *Taccuino senese*, a sketchbook in the collections of the Biblioteca Comunale degli Intronati in Siena).[4] Such studies may provide scholars with opportunities to understand more fully the processes and intentions of individual architects. Parametric modeling allows for exceedingly specific analysis, while digital systems facilitate the collection and arrangement of data necessary for describing the geometry of all elements in a work of architecture.[5] In this article, we propose a new hypothesis regarding the design of Santa Maria delle Carceri based on our research using 3-D modeling; we also offer new perspectives concerning Sangallo's design system.

The Church of Santa Maria delle Carceri

Santa Maria delle Carceri in Prato holds a significant place in the history of architectural typologies because of its centralized plan (Figures 3 and 4). Thanks to the work of architectural historians Piero Morselli and Gino Corti, the building's construction history is well known, and we are able to determine the role of drawings and models in the Renaissance design process with unusual precision.[6] Archival documents also provide valuable information about the influence of the building's patrons on its design and construction. Architectural historians such as Giuseppe Marchini, Paul Davies, Riccardo Pacciani, and Claudio Cerretelli have examined many different aspects of the building and its history, including the institutions involved, its urban setting, its construction and architectural details, and the geometric system underlying its design.[7] In his 2016 study of centralized Renaissance churches, Jens Niebaum considers the place of Santa Maria delle Carceri within the typology of the pilgrimage church, linking it to religious symbolism and the revival of classical antiquity.[8] The current investigation builds on these previous studies by offering a precise analysis of Sangallo's modular design system.

Centralized structures with clear geometric patterns—circles, squares, octagons, polygons—fascinated Renaissance humanist patrons and architects. Soon after construction began at Santa Maria delle Carceri, Leonardo da Vinci, perhaps influenced by Donato Bramante, developed a now-famous series of drawings (Manuscript B held in the Institut de France, Paris) showing many variations on the church's centralized plan.[9] Centralized layouts, however, often failed to meet the needs of the liturgical functions of the Christian church, whose large congregations and complex, dynamic modes of worship required longitudinal naves. Monuments such as the Florence Baptistery, the Basilica of Our Lady in Loreto, and Alberti's San Sebastiano in Mantua represented grandiose yet practical adaptations of the central plan to

Figure 4 Giuliano da Sangallo, Santa Maria delle Carceri, Prato, begun 1485, aerial view with the castle of Frederick II of Hohenstaufen in foreground (image from Microsoft Bing Maps, 2017, edited and customized by the authors).

Christian liturgical requirements.[10] Functionally speaking, the central plan was particularly well suited for oratories, sanctuaries, and mausoleums; notable examples include Filippo Brunelleschi's Santa Maria degli Angeli in Florence and Giuliano da Sangallo's Santa Maria dell'Umiltà in Pistoia. Pilgrimage churches dedicated to the Virgin Mary featured centralized plans with some frequency. Since the early seventh century, the Pantheon in Rome had been dedicated to Santa Maria della Rotonda, or Santa Maria ad Martyres, beginning a tradition of centralized buildings used to express devotion to the Virgin Mary; later examples included Brunelleschi's Santa Maria degli Angeli in Florence and Gian Lorenzo Bernini's Santa Maria Assunta in Ariccia.[11] In Prato, the miraculous apparition of the Virgin Mary to eight-year-old Jacopino d'Antonio in July 1484 led to the inauguration of Santa Maria delle Carceri.[12]

One year later, in 1485, the municipal authorities of Prato chose a design for the church by the Florentine architect Giuliano da Maiano.[13] As Paul Davies has noted, the size and shape of da Maiano's proposed design, recalling the Florence Baptistery, cannot be reconciled with the existing building's footprint.[14] In addition, this design would have transgressed the restriction imposed by the Dieci di Balìa, the Florentine magistracy in charge of military affairs, stipulating that the new church had to stand at a distance of at least 24 *braccia* from the nearby castle of Frederick II of Hohenstaufen.[15] In any event, da Maiano's proposal reveals the important evolution of such typologies with its inclusion of a large portico surrounding the church, where pilgrims could gather and have indirect access to liturgical functions through the windows. Lorenzo de' Medici, who followed the project closely, was not convinced by this choice and halted construction soon after the foundations were erected. Prato was under the control of the Republic of Florence at the time, and therefore the Dieci di Balìa strongly supervised the dimensions of the church's placement. At this point, Lorenzo became personally involved in the project, which coincided with the construction of his famous villa at Poggio a Caiano by his architect Sangallo, who synthesized local tradition with classical models. In September 1485, Lorenzo read chapters of Alberti's treatise *De re aedificatoria* and also ordered a survey or a model of Alberti's centralized Greek cross design for San Sebastiano to be delivered personally to Lorenzo in Florence by Luca Fancelli, the architect charged with the execution of Alberti's buildings in Mantua.[16]

However, it seems another Tuscan pilgrimage church, the Madonna della Pietà at Bibbona, where a similar apparition of the Virgin occurred some years earlier, inspired the centralized plan selected for Prato.[17] Here, Vittorio Ghiberti, the son of the famous sculptor Lorenzo Ghiberti, also constructed a church according to a Greek cross plan, but on a much smaller scale. This form was in fact more suitable for expressing the hierarchy between the center and the subordinated spaces and, in the case of Prato, for affirming the significance of the church as both a religious and a civic symbol. The dimensions of Santa Maria delle Carceri, with a central square measuring 20 *braccia* per side (11.72 meters) and arms half that length (10 *braccia*, or 5.84 meters), try to conciliate with the dimensions of the nearby fortress in the surrounding urban landscape.[18] Sangallo's use of clear geometric forms, placing a dome above a cube, not only emphasized the midpoint of the structure but also contributed to its urban significance. Like the dome of Brunelleschi's Sacrestia Vecchia, the dome of Santa Maria delle Carceri is covered by a sloping roof and thus not visible on the exterior. The design of the Sacrestia Vecchia also evidently inspired Sangallo's decorative pilaster order on the interior.

When Sangallo left the building site in 1499, the façades were not completed, and the upper exterior order was completely missing. The construction of the upper section of the southwest façade, including its Ionic pilasters and pediment, occurred in the period 1884–87. As we will see, this nineteenth-century design is not entirely consistent with Sangallo's design principles.

Analyzing Santa Maria delle Carceri Using New Technology

The only authentic graphic source regarding Santa Maria delle Carceri, probably from Sangallo's own hand, is a plan in the *Taccuino senese* (Figure 5). The parchment sheet represents the building in different forms and thus is of an uncertain date, but details like the niches in the inner walls, the apse, and the pilasters flanking the doorways, later eliminated by the architect, suggest it predates the construction of the church. The freestanding volume reflects an idealized design that ignores the preexisting prisons to the north. Nevertheless, the plan approximates the final solution in which Sangallo accommodated local restrictions (preserving the existing prisons, one of which contained a miraculous painted image) while maintaining the distance from the nearby castle as required by the Dieci di Balìa.[19] On the other hand, on the upper part of the same page in the *Taccuino senese* there is also a plan of the Chapel of Sant'Aquilino at the Early Christian Basilica of San Lorenzo in Milan, believed to have been drawn by Sangallo during his stay in Milan in 1492, suggesting that the plan of Santa Maria delle Carceri can be dated to the same year. We know nothing about Sangallo's *Wanderjahre*, and he may have previously visited the capital of Lombardy, where his Florentine colleagues Leonardo da Vinci and Bramante conceived and realized innovative architectural projects. However, it is not easy to understand why Sangallo, or a later copyist, would have produced an obsolete design, given that a new version of the plan was already under construction. Did the architect wish to document ideas he had developed before, either for his own use or for his *bottega*? Surprisingly, this may be the case, given that some options (i.e., the rhythm and the harmony of the architectural order of pilasters) in the actual building are more convincing and better reflect the evolution of Sangallo's architectural language.

The verso of the same sheet, which includes a plan of the Villa di Poggio a Caiano, likewise reveals details of an earlier project.[20] Unfortunately, we are still unable to determine a precise chronology, as technical analysis of the parchment sheet has not resolved the dating issues. Even if the drawing in the *Taccuino senese* cannot be considered an original scheme, some details shed light on other ideas formulated by the architect early in the design process. By comparing this drawing with a recent survey of the pilgrimage church produced using laser technology, we are able to reconstruct and verify those preliminary ideas.[21] Our primary purposes in

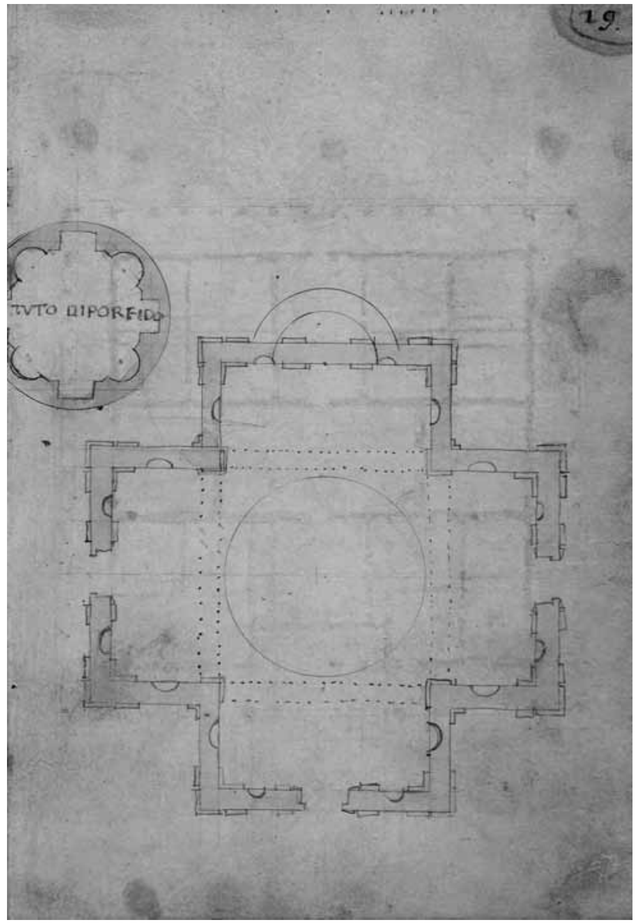

Figure 5 Giuliano da Sangallo, plan of Santa Maria delle Carceri, Prato, date unknown (*Taccuino senese*, fol. 19r, ms. S. IV. 8, Biblioteca Comunale degli Intronati, Siena).

undertaking this survey were to analyze the relationships between the existing monument and Sangallo's plan, to better understand the drawing's graphic construction, and to reconstruct possible solutions and variants beyond those indicated in the drawing itself. Our study does not address problems related to site limitations at Santa Maria delle Carceri, such as the preexisting fortress and prisons to the north (discussed by Cerretelli), or Sangallo's sources for the church's architectural order and decorative elements (investigation of these would require further examination of Sangallo's drawing corpus, including the *Codex Barberini* and the *Taccuino senese*).[22]

The centralized plan found in the *Taccuino senese*, conceived according to *braccia fiorentine*, is inscribed within a square like the built pilgrimage church itself, following proportions derived from Alberti's *De re aedificatoria*.[23] Consequently, the arms, spanned by barrel vaults, measure half the width of the square in the center crowned by the cupola. The wall mass, drawn at a dimension insufficient to support the weight of the drum and the cupola, was later thickened and reinforced.

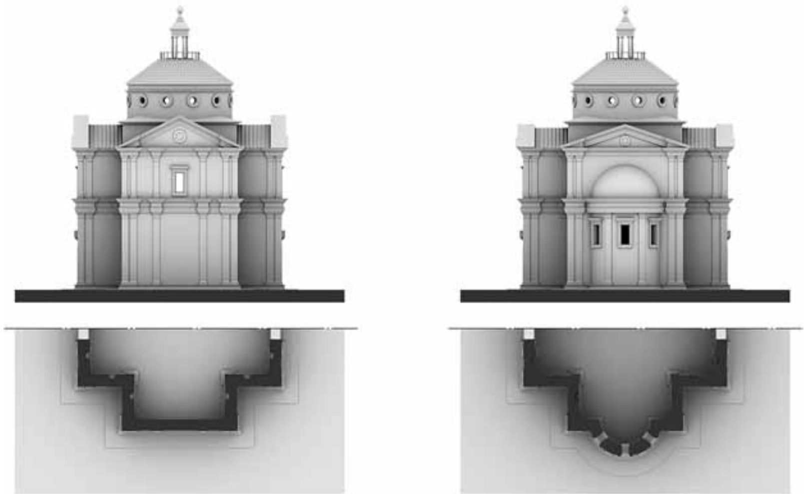

Figure 6 Two hypothetical reconstructions of the sanctuary of Santa Maria delle Carceri, Prato: the regular Greek cross plan (left) and the same plan with the addition of a segmental apse (right) (images by the authors).

The drawing shows two schemes. The first is a regular Greek cross with four identical façades (Figure 6). The second includes a semicircular apse at the rear of the building. As Davies has demonstrated, the proposed apse interfered with the ruins of the prison and the miracle-working image located there.[24] Given that Prato's civic council required the preservation of both these preexisting elements, it is unlikely this scheme would have been accepted.

The use of semicircular niches carved into the thickness of the walls recalled ancient Roman architectural motifs.[25] Sangallo also employed niches in his earlier design for Palazzo Scala in Florence, but they were of a shallower profile than the semicircular recessed niches of the *Taccuino senese* drawing. The façade design shown in the sketch differs from the actual building, with two additional pilasters placed on either side of the door at the center of the three main façades, both inside and out. The portals also do not appear as they were actually executed.[26] These details in the drawing introduced a greater vertical emphasis that would have significantly modified the exterior appearance of the church. As executed, the façade surface extends unobstructed between the reinforced corners, suggesting a more mature design solution.

The reconstruction of the upper order of the façades poses further challenges, as here we have absolutely no indication of Sangallo's intentions. According to both Renaissance principles and Vitruvian rules, which require a structural relationship between roof and gable, the pediment as executed is too high. It is unclear whether Sangallo meant to use Ionic pilasters or whether he intended to adorn the second level with a more abstract attic order. Regardless, the virtual reconstruction reveals the symmetry and homogeneity of the architectural framework developed by Sangallo and his adviser Lorenzo de' Medici to introduce pilgrims and believers to the miracle.

The reconstruction proposed here is intended to enable a fuller understanding of these early phases of the design, including the ideas of Lorenzo de' Medici, who probably desired a coherent organism based on Alberti's canon of harmonious proportions and a perfect structural correspondence between the inner and exterior orders.[27] We also discuss below how the digital model clarifies Sangallo's use of a modular system and his attention to architectural details such as the design of the internal entablature, with its three constituent components each measuring 1 *braccio* in height. Such a study enables us to better understand the design process behind Renaissance architecture, where most projects corresponded to ideal proportional systems, although often compromised by physical and functional exigencies. The model presented here can also help us to reconstruct the architect's design methodology and the construction practice. Masons often relied on local building traditions, thereby introducing further variations to architects' designs. However, the virtual reconstruction of Santa Maria delle Carceri reveals an astonishing correspondence with the *Taccuino senese* drawing, indicating that Sangallo managed to maintain exceptional control over the construction process. For this reason, the structure holds an outstanding place in the history of late quattrocento architecture.

Thanks to the level of detail afforded by the virtual model, we can also gain a better understanding of the gradual development of solutions and alternatives proposed for Santa Maria delle Carceri. Some details, like the design of the apse windows and the order of the exterior upper story, must remain hypothetical. In such cases the virtual model integrates contemporary architectural motifs to generate a coherent design. The sophistication of this complex new technology opens up new areas for research and has the potential to deepen our formal and technical understanding of Renaissance architecture.

3-D Model Construction and Data Analysis

Today's architects and architectural historians are familiar with the use of 3-D models as a means to reconstruct missing elements in existing buildings or to construct unbuilt structures virtually. Until now, practitioners and scholars have largely used this technology, which has been in development for more than twenty years, to translate measurements into photorealistic images, either to verify proposed design solutions or to visualize the overall appearance of particular buildings. Used for these purposes, even if 3-D renderings offer new ways to animate or quantify architectural spaces, they do not provide much information beyond that already available in 2-D drawings.

Despite many recent experiments, few 3-D models have been created using the appropriate semantics for the architecture under analysis. While the association of semantics and architectural shapes may facilitate a building's examination as a cognitive system, the potential mismatch between semantics and these same architectural shapes may also make it more difficult to distinguish the architect's original intentions for the project from later modifications.[28]

To investigate Sangallo's design process for Santa Maria delle Carceri, we employed a method that involved generating a digital representation of the building as it currently stands and then superimposing deficiencies, irregularities, modifications, additions, deletions, and restorations as these occurred over time. We also compared what we found with data generated through critical investigation of Sangallo's drawings as well as other Sangallo buildings, buildings by Sangallo's contemporaries, and surviving visual and textual references, including Alberti's *De re aedificatoria*. One of our key goals was to determine whether the *Taccuino senese* plan and its embedded proportional system carried into the design's subsequent development.

The technique we used relies on a specific semantic 3-D-based information system to build a knowledge framework that can organize surveyed data; verify measurements, proportions, shapes, and constructive solutions; and develop photorealistic reconstructions to test alternative proposals. Drawing upon recent digital innovations, the workflow uses both building information modeling, or BIM, and knowledge-based modeling to translate semantic compositions and their relationships into 3-D models.[29] We developed specific semantics based on elements drawn from two key Sangallo sources: the *Codex Barberini* and the *Taccuino senese*.[30]

Developing these 3-D models involved translating between two systems of knowledge. Sangallo's descriptions of tangible elements can be used to generate BIM components and parametric models using constraints based on Sangallo's module. Instead of drawing specific shapes, we can replace dimensions with variables and set formulas. All dimension adjustments are automatic, so that any given modification is reflected across a model in real time.

Our process involved four ideal steps: (1) BIM-based, knowledge-based modeling; (2) data capture and as-built model construction; (3) ideal and as-built model integration; and (4) multidimensional analysis (Figure 7). Through knowledge-based modeling, the geometric relationships between architectural components specified in architectural treatises and other theoretical sources can be integrated into a BIM system. The resulting BIM provides a template that collects and orders surveyed data. Unlike traditional computer-aided design, or CAD, systems, which require intensive drawing, BIM manipulates the existing survey data by controlling parameters.

The data-capture approach integrates both range-based and image-based modeling. Single capture techniques do not provide satisfactory results for all situations (e.g., when there is a need for high geometric accuracy, portability, automation, photorealism, low cost, flexibility, or efficiency). By combining image and range data, the BIM system can fully exploit the intrinsic potential of both approaches. Terrestrial laser scanning, or TLS, can be integrated with automatic photogrammetry to obtain the extremely accurate images essential to 3-D modeling (Figure 8).[31] Photogrammetric-based 3-D model construction, a completely automated preprocessing procedure, provides a robust level of image matching that uses both raw image files and color photographs (Figure 9).

Following the integration of the ideal model generated from the parametric library and the as-built model obtained from the captured data, any deviations are evaluated to determine if feature extraction from point clouds may improve the accuracy of the as-built BIM. Full automation from point clouds to the as-built BIM is not yet possible using existing software, but in most situations it is not necessary to document all the as-built data, such as irregular shapes, with BIM. The process, which we applied to analysis of the interior entablature of Santa Maria delle Carceri, comprises the steps described below (Figure 10).

Adjustment of BIM objects to point clouds: Knowledge-based modeling provides a parametric template that can be integrated with data sourced from the as-built condition. The BIM building elements from the parametric libraries are adapted to the point clouds. If the building element family already exists in an object library, a new type is created through the adaptation of the parameters of the existing family to the point clouds; if not, a new building element family is created and added to the parametric library for future use. In practice, the average dimensions of the BIM elements are obtained from the captured 3-D data. Data generated by the comparison of these measurements in turn allow for the parameterization of the object based on relationships.

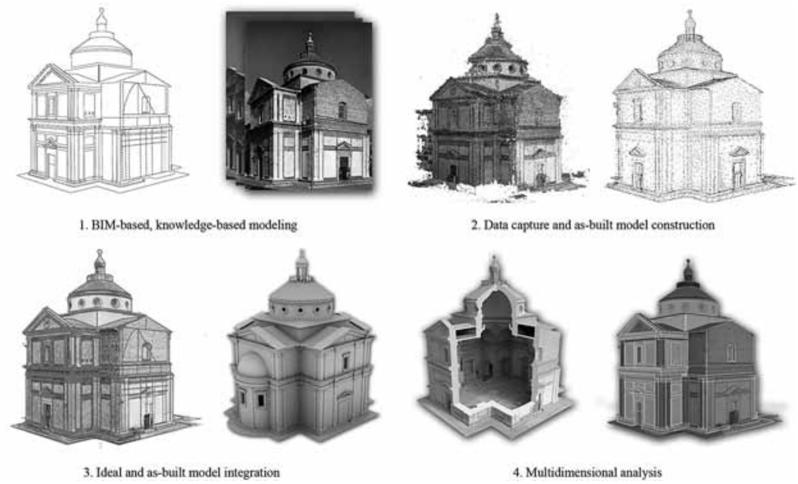

Figure 7 Four-step sequence of the semantic 3-D-based information system used to examine Santa Maria delle Carceri, Prato (images by the authors).

Figure 8 Data capture using terrestrial laser scanner: Santa Maria delle Carceri, Prato, plan view and dense point cloud of the front elevation (images by the authors).

Figure 9 Data capture using photogrammetry: Santa Maria delle Carceri, Prato, Doric base (images by the authors).

Evaluation of deviation: Existing building components are prone to irregularities because of damage, deterioration, and deformation over the course of their life cycles. Although the BIM objects are mapped on the captured point clouds, the graphic primitives in BIM may not be sufficiently precise when the deviation reaches a certain threshold. Consequently, we used a tool to automatically evaluate such deviations and identify objects that failed to meet the demanded accuracy. Such objects were instead modeled using the extracted features of captured point clouds.

Feature extraction: Various approaches may be used to topologically bridge the gap between point clouds and BIM. For example, cross-sectioning is an effective method for extracting features from point clouds to BIM, since the overall design and proportions of classical architecture are closely related to the shapes and proportions of individual moldings. Most of Sangallo's architectural elements can be modeled through the sweeping of certain profiles along straight lines or curves. This is not merely a coincidence; rather, it reflects the Renaissance craftsman's technique of using a template to make a profile on a block of stone before cutting away the excess. Cross sections can be transformed into 2-D drawings and saved in BIM as reusable profile families nested in other families.

Finally, multidimensional analysis facilitates the extraction of many insights from produced outputs and simulations. BIM-based 3-D modeling allows multiple outputs that can exploit multidimensional and multisensory analysis techniques, including not only simple renderings and computer graphic photorealistic animations but also physical models generated using 3-D printing technology and virtual-reality

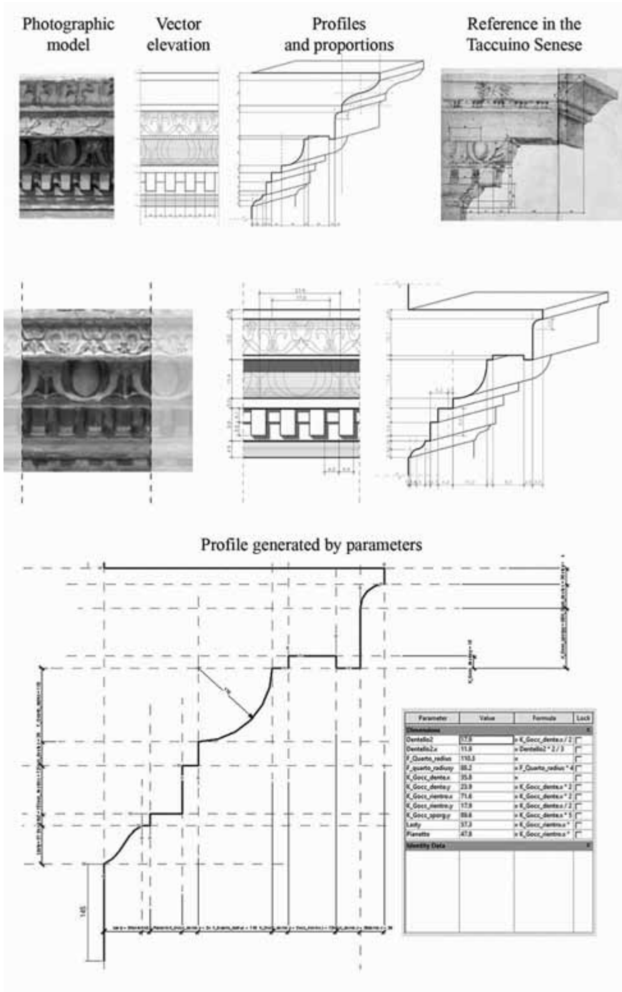

Figure 10 Ideal and as-built model integration: Santa Maria delle Carceri, Prato, interior entablature (images by Carlo Bergonzini, edited and modified by the authors).

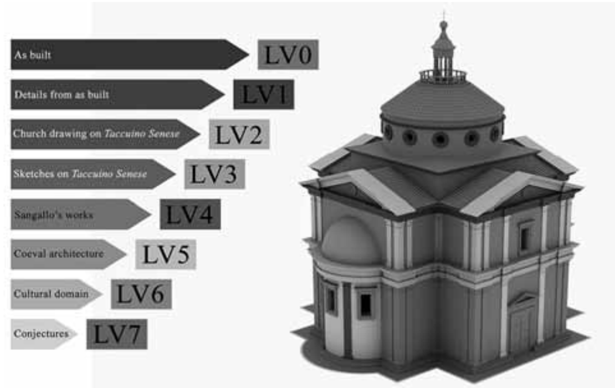

Figure 11 Different levels of interpretation/uncertainty regarding reconstruction of Santa Maria delle Carceri, Prato, visualized through a pseudo color code (image by the authors).

Figure 12 Multidimensional analysis using 3-D models: assessing construction solutions at Santa Maria delle Carceri, Prato, through digital fabrication (image by the authors).

visualizations that make use of perceptive cues as well as false color visualizations highlighting those aspects that might otherwise escape notice. The aim is to show not just the complete interpretational process but also the reliability of its different components.

The solution involves eight levels of reconstruction, characterized by a progressively increasing uncertainty visualized through a pseudo color code that divides the rendered objects into a small number of color bands corresponding to a hierarchy based on level of interpretation or uncertainty (Figure 11). Thus level 0 (LV0) includes elements from the as-built church; level 1 (LV1) extends actual details of Santa Maria delle Carceri to other parts of the building; level 2 (LV2) includes building portions inferred from the *Taccuino senese* drawings; level 3 (LV3) represents elements following other details from the *Taccuino senese* sketches; level 4 (LV4) corresponds to solutions found in other Sangallo works; level 5 (LV5) describes elements inspired by other contemporary buildings belonging to the same typology; level 6 (LV6) refers to architectural elements sketched by other contemporary architects; and level 7 (LV7) includes conjectural elements without specific references.

Finally, 3-D models generated using 3-D printing techniques allow for further multidimensional analysis (Figure 12). The process involves qualitative and quantitative methods for evaluating and testing the design process and the resulting model. In addition, 3-D parametric descriptions and rules concerning materials and material assembly provide a new construction grammar. These construction rules do not just determine the visual appearance of building components, they also define the components' assemblage in 3-D space, providing further structural and functional information about the building.

Reconstructing Santa Maria delle Carceri

As noted, our investigation of Sangallo's design for Santa Maria delle Carceri, including both built and unbuilt components, involved a multiscale, dual-source process based on an analysis of the *Taccuino senese* drawings and their relationship to the church as it stands today, as well as a new, accurate survey of the existing structure.

Investigating the Taccuino senese for BIM-Based, Knowledge-Based Modeling

We began our investigation with an analysis of Sangallo's note-taking process as well as an analysis of the measurements used to generate the *Taccuino* plan. Sangallo's notebook includes numerous combinations of textual and numerical symbols, following architectural conventions still used today, with dimensions presented as text alongside annotation lines to generate a measurable communicative system. Starting with Sangallo's most legible annotations, we first verified these symbols against their corresponding lengths in the real domain and then used them to check the other dimensions lacking corresponding measurements in Sangallo's drawings. We were able to do this by identifying the largest element with a simple symbol, assigning it a value from the measured survey of the existing church in Prato, and then scaling the drawing using computer-aided drafting software (given that at this stage the units of measurement were unknown, we initially adopted a relative scale). Finally, we measured and checked the values of other annotations against the measured real value, while acknowledging the inevitable variations and inaccuracies resulting from a hand-drawn rendering as well from a sheet that was perhaps not perfectly planar (Figure 13).

Comparison of the measurements of the drawings in the *Taccuino* to the built architectural spaces confirmed that as a unit of measurement Sangallo employed the *braccio fiorentino* (one *braccio* = 0.5836 meters).[32] Further study of the design details revealed that he also made frequent use of submultiples of the *soldo* (0.0292 meters) and the *denaro* (0.0024 meters).

Assuming that Sangallo followed this dimensioning scheme, we tested eight order elements represented on folio 12v of the *Taccuino* and found deviations of, at most, 13 percent (in one case). Most of the verified comparisons stood within the threshold of 8 percent, thus confirming our hypothesis that Sangallo used the *braccio fiorentino* as a standard unit of measure. Following this method, we assimilated the individual design components of the *Taccuino senese* to generate a real measurement scale. With this as our unit of measurement, we analyzed the ground plan of Santa Maria delle Carceri and found that it reveals interesting details.[33] To begin, the top right side of the small sheet of brown-inked paper features a set of reference points, subdivided into twenty parts. As the internal square of the church measures approximately 20 *braccia fiorentine* (11.72 meters), it follows that each subdivision equals precisely 1 *braccio*. A further linear division generates five exact submultiples, each with a distance equal to 1 *braccio*. This metric scale allowed us to understand the relationships governing Sangallo's various architectural components. Transferring these reference lines to the Sangallo plan revealed how he ordered and proportioned the wall profiles and interior spaces (Figure 14).

Sangallo's design highlights simple layouts, privileging proportional relationships between squares and circles of 1:1 and 1:2. Such modules govern every aspect of the plan. By taking this square as a base module of M (M = 5,836 meters, for which the general side is equal to 10 *braccia fiorentine*), we can trace all the measurements back to the module. In plan, the diameter of the dome is identical to the width of the lateral wings and double their depth. (As previously noted, the boundary wall thickness is roughly a quarter of M, or 1.459 meters.) The proportions we discovered inside the pilgrimage church also regulate its exterior; thus, the lateral façades of the wings follow the M modularity, distinguished by a 1/4M separation from the frontal façade (Figure 15).

We created a semantic data model built as a cognitive BIM system using *braccia*, *soldi*, and *denari* as units. This in turn allowed us to systematically verify the local and global proportions of the building.

Constructing an As-Built Model to Compare with Existing Surveys and the Taccuino senese

Surveying the actual building, both inside and out, represented a critical phase of the project, as it allowed us to compare elements inferred from the general plan view in the *Taccuino senese* with the building's constructed dimensions. Further comparison with existing drawings also generated many novel insights.

To complete the survey of the building, inside and out, we conducted twenty-six laser scans. Using Faro Scene 5.0 software, we created a general reference point cloud relating to Santa Maria delle Carceri, processing color raw files collected with a Focus3D Cam2 phase shift laser scanner (this device features a 3-D point accuracy of up to 2 millimeters; working distance = 0.6–150.0 meters). The ground sample distance, or GSD (the true distance between the sampled points), could vary between 4 and 15 millimeters, depending on whether the captured data referred to the whole church or only to certain parts of the building.

The photogrammetric 3-D model was assembled from 230 images taken with a Canon EOS 1000D SLR camera.[34]

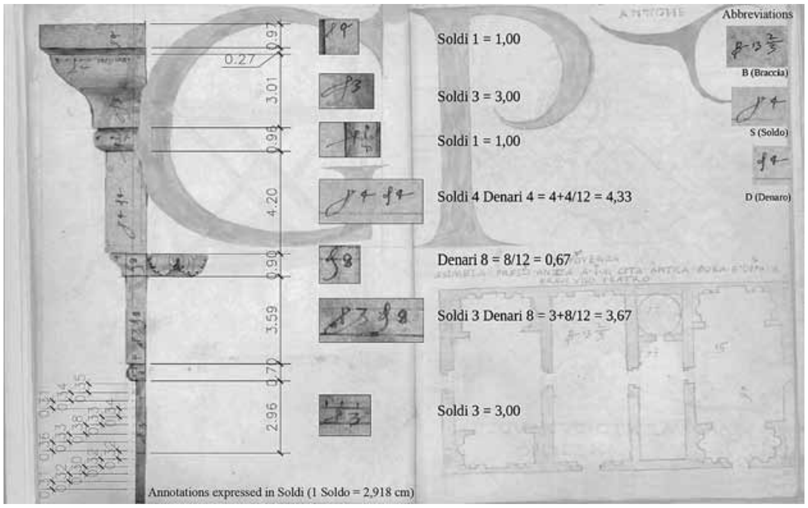

Figure 13 Santa Maria delle Carceri, Prato, analysis of measurements using folio 12v of the *Taccuino senese* (ms. S. IV. 8, Biblioteca Comunale degli Intronati, Siena).

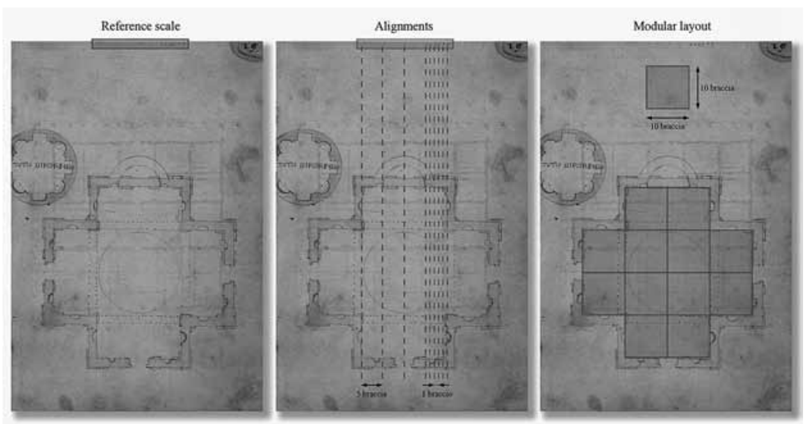

Figure 14 Santa Maria delle Carceri, Prato, analysis of plan proportions using folio 19r of the *Taccuino senese* (ms. S. IV. 8, Biblioteca Comunale degli Intronati, Siena).

Selected images were characterized by their distance from an object included at a range of 4 to 16 meters, taken with a 35-millimeter focal length. The GSD could vary between 0.5 and 2.0 millimeters, allowing for a more accurate documentation of architectural details than that offered by the laser scanner survey.

This process enabled us to generate a faithful measured reproduction of the existing structure. By analyzing this evidence, we could better evaluate the potential dissimilarities from the drawing, mainly regarding the building's upper parts. Measurements registered within an accuracy of 1 centimeter for the entire building, reaching submillimeter accuracy for architectural details, ensuring consistency in further multidimensional analysis.

Next, we compared this survey to the measured drawings by Morselli and Corti, who performed a manual survey based on total station, or TS, reference data in 1982.[35] While our survey resembled Morselli and Corti's drawings, especially in the lower parts of the building, there were discrepancies in height estimates, especially concerning the positioning of the cornice of the dome's exterior. In plan, these differences were minimal: for example, our measurements for the width of the church varied by only a few centimeters. Major deviations emerged, however, when we compared vertical sections, such as the overall height of the building (the sphere of the lantern is, in effect, lower by approximately 0.40 meters, while the vault's base is higher by approximately 0.50 meters). This is a particularly important point, since our survey revealed some key differences regarding certain features recorded in 1982. As demonstrated by our recent laser scans, the dome and the lateral vault present sections with different profiles. Neither of the profiles is circular, as was indicated in the 1982 survey. Instead, the profiles are more elongated ovals (Figure 16).

Integrating Ideal and As-Built Models

We compared and superimposed point clouds and elements collected during the survey onto the plan from the *Taccuino senese* to verify and adjust the proportions of individual elements and their integration with each other. In this way we produced an ideal parametric 3-D model. The later survey of the existing building proved our findings to be accurate (for

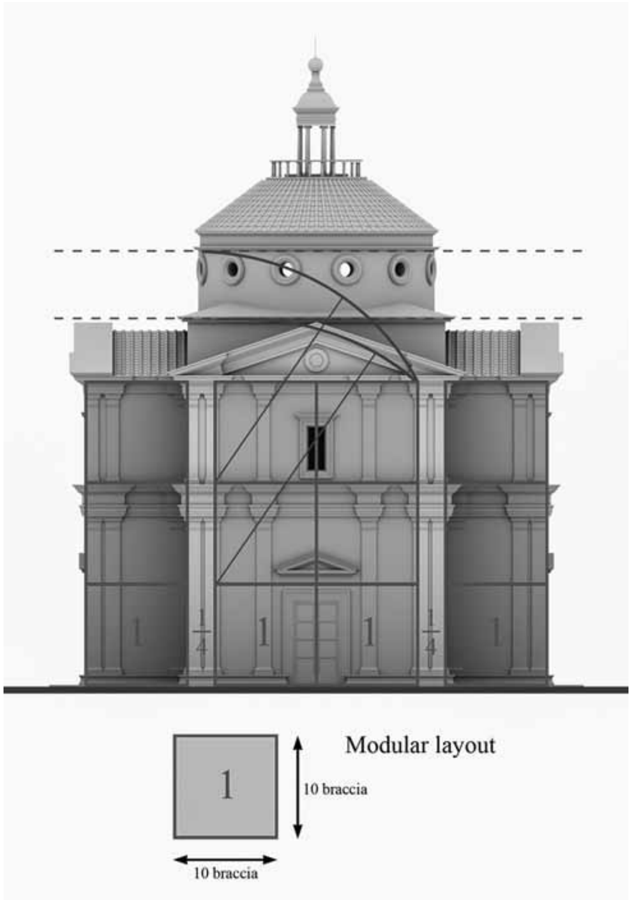

Figure 15 Santa Maria delle Carceri, Prato, analysis of elevation proportions derived from the integration of ideal and measured 3-D models (image by the authors).

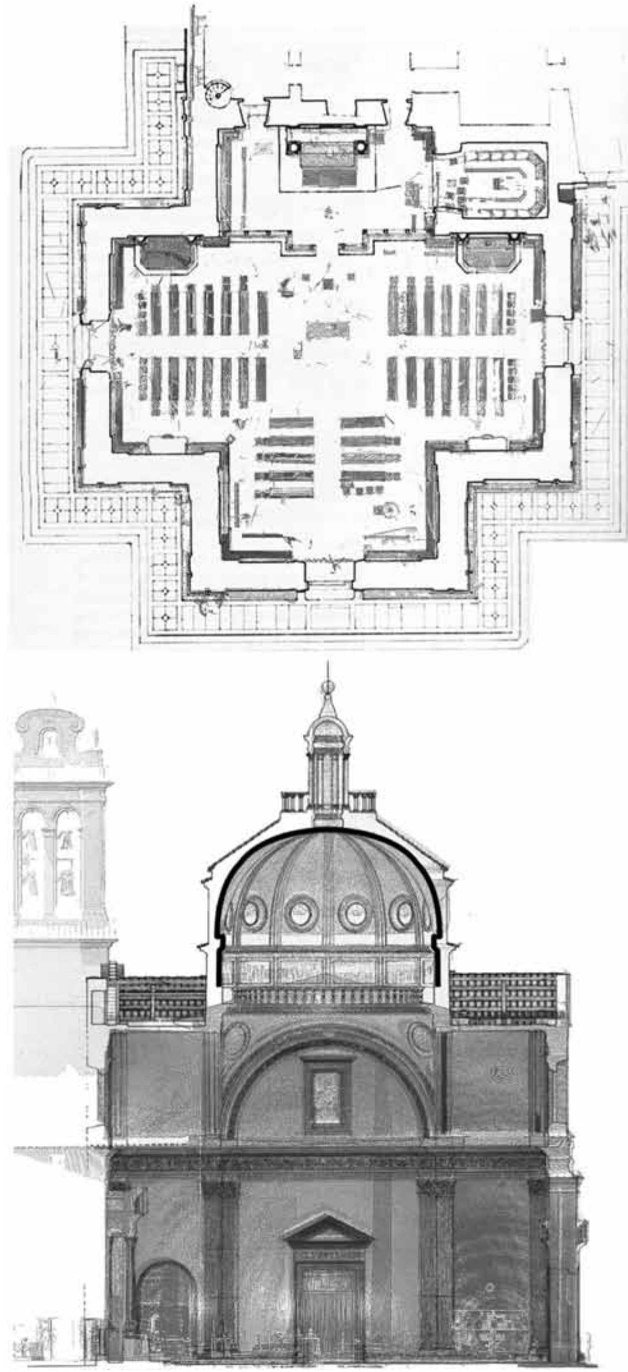

Figure 16 Santa Maria delle Carceri, Prato, plan and elevation generated by 3-D survey superimposed on Morselli and Corti's 2-D survey of 1982 (Piero Morselli and Gino Corti, *La chiesa di Santa Maria delle Carceri in Prato: Contributo di Lorenzo de' Medici e Giuliano da Sangallo alla progettazione* [Florence: EDAM, 1982]; images by the authors).

example, the surrounding pavement from which the church rises is generated by a shape mostly equal to one-half of the base module M).

We used the same module and method to verify that Sangallo adopted the same proportions for the exterior as for the interior. The architect seemed to favor similar dimensions for the vertical development of the church: the height between the floor and the entablature is equal to two base modules, or two M, the distance between the springer and the top of the barrel vault is equal to one M, the dome is one M high, and the walkway is half an M (see Figure 15).

Our analysis verified that the harmonious proportions of the interior correspond to those of the external preserved façade, as observed by Maurizio Filiaggi.[36] The height of the entablature's upper ends, both internal and external, which corresponds to two M, confirms a logical consistency between the interior and the exterior.

In addition to identifying the modular proportions linking interior to exterior, our analysis verified that Sangallo made use of the golden ratio to locate various elements, such as the base of the drum and the upper limits of the twelve oculi, or circular windows, above. This system determined the positioning and heights of the two central entablatures and the various heights of the portal elements in the façade.

The entire design logic of the building is driven by the proportions of the square pilasters placed at its corners, each measuring 1.5 *braccia fiorentine* wide and 16 *braccia* high (or 1:10.333). These proportions are more elongated than the canonical 1:7 or 1:8 ratio of the Doric order. Sangallo's drawing on folio 31v of the *Taccuino senese* explains this customized grammar, subdividing columns into twenty or thirteen parts (Figure 17). This logic is also evident in the actual building. For example, the height of each square pilaster's base is one-twentieth of the column's height, while the capitals are one-thirtieth the height of their columns (measured from the top of the column to the top of the collar). The two central bands and the various heights of the portal elements also conform to this principle. All three elements of the entablature—architrave, frieze, and cornice—are the same height, each precisely 1 *braccio* (Figure 18).

A section generated from our point cloud facilitated comparison between internal and external pilaster shafts, even if overall dimensions reveal substantial differences. Sangallo adhered to the principle of dividing the height of each pilaster into twenty parts, as a rule that determined both the height of its base (1/20) and the height of its capital (2/20). The heights of the pedestals are less consistent: the pedestal bases in the interior are lower than those on the exterior.

While the capital and overall entablature height reflect a clear correspondence between interior and exterior, the various elements do not always correspond exactly. For instance, while both the inner and the outer entablature are equally divided into three parts, the inner entablature extends to a slightly greater height to support the spring line of the barrel vaults.

Overall, the flexible framework of the *braccio fiorentino* facilitated Sangallo's dimensioning of partitions and placement of decorative features. Throughout the project, from the general scheme to the design of individual elements, Sangallo employed a clear and consistent design methodology.[37]

Hypothetical Reconstruction Derived from Proportions

In our reconstruction of the existing, incomplete church, we relied on the *Taccuino senese* plan to complete the upper portions of the exterior and to analyze interior variations. We created a semantic data model that retained original elements and removed subsequent alterations and additions, such as the internal balustrade in front of the altar and the aediculae adjacent to the nave.

Comparing the drawings in the *Taccuino senese* to the actual structure helps to illuminate many subsequent modifications. The proportions and measurements that we documented in Prato suggested some hypothetical answers to questions regarding Sangallo's intentions for the façades and the interior. For example, as noted earlier, Sangallo's plan in the *Taccuino* shows additional external pilasters flanking the main portals and a semicircular apse on the northern façade.

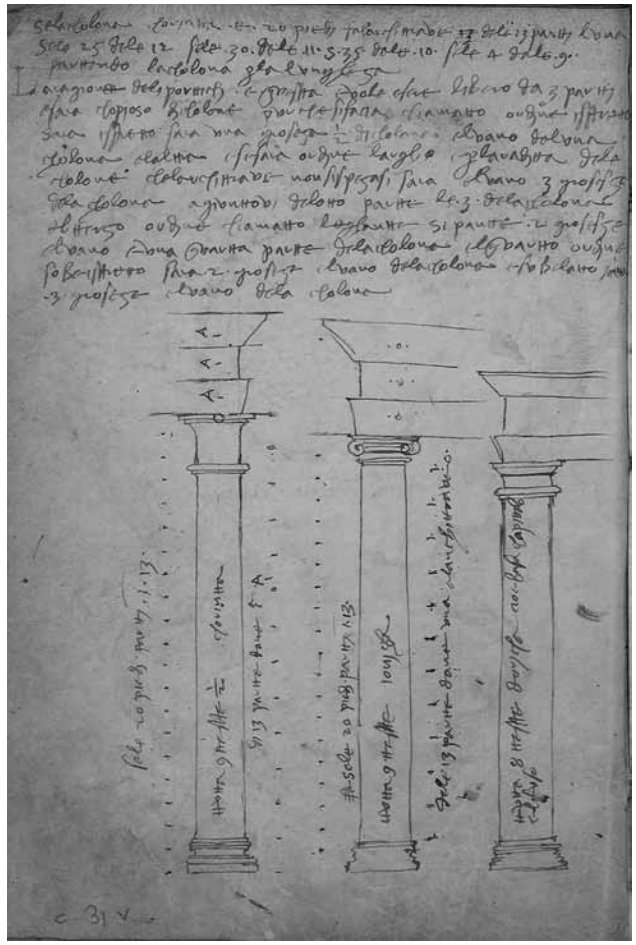

Figure 17 Giuliano da Sangallo, column proportions, date unknown (*Taccuino senese*, fol. 31v, ms. S. IV. 8, Biblioteca Comunale degli Intronati, Siena).

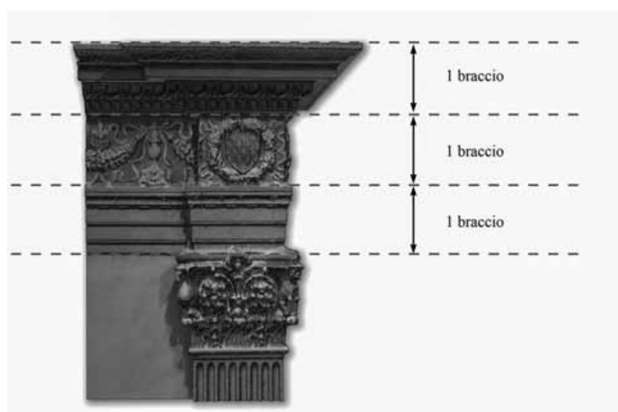

Figure 18 Santa Maria delle Carceri, Prato, entablature proportions as generated through the integration of ideal and measured 3-D models (image by the authors).

Our digital models for Santa Maria delle Carceri addressed the exterior and interior separately. For the outer elements, we first retraced the plan of the *Taccuino senese* as a vector digital drawing. We then prepared a preliminary boundary by extruding walls from the vector drawing, integrating distances and alignments. We modeled the first architectural order in elevation, taking into account the previously conducted survey (to ensure consistency with the heights of the actual building), and then applied moldings, profiles, and decorations to replicate the exterior surface ornament, including the double pilasters placed at the corners. The dimensions of the pilasters flanking the portals as shown in the *Taccuino* duplicate those of the existing pilasters at the corners, while their spacing corresponds to the original drawing.

In fact, in the *Taccuino senese*, Sangallo provided a comparison of architectural orders, following a well-defined proportional scale. Those elements in turn offered a cue for the second architectural order. Drawing upon the evidence from the survey, we proposed a hypothetical second order, modifying its proportions to fit with the existing Doric composition.[38]

The reconstructed pediments follow the Vitruvian recommendation that these must coincide with the height of the roof.[39] Our design took as an example the tympanum Sangallo placed above the portico at the Villa Medici at Poggio a Caiano (despite its uncertain dating).[40]

The existing façade provided the model for the remaining missing façades, consistent with the symmetry of the centralized plan. The extruded drum corresponds to the hatched tracing in the *Taccuino* drawing, from the elevation recorded in the survey to the circular vault with its twelve oculi. Finally, the design of the lantern corresponds to its actual form as built, as confirmed by a photogrammetry-based survey we conducted from the adjacent fortress of Frederick II.

As Sangallo provided an alternative solution in the plan in the *Taccuino senese*, we also added a semicircular apse to the northern façade of the digital model. In accordance with Sangallo's traced perimeter, we proposed three hypothetical openings to illuminate the apse, corresponding to the existing proportional system. Both the design of the openings and the external order replicate those elsewhere on the church, maintaining a consistent architectural language. Pilasters subdivide the exterior of the apse into three identical bays, corresponding to the pilasters ornamenting the exterior walls as shown in the *Taccuino* plan (Figure 19).

As for the interior, we found that the wall profiles and structural grids of the plan shown in the *Taccuino* also dovetailed with the design of the existing church. Our detailed survey allowed us to accurately model the existing pilasters, the supporting barrel vaults, and the dome. The dimensions of the pilasters at the corners correspond to those of the existing design, documented using an accurate photogrammetric modeling process. These same pilasters, added later by Sangallo, provided the prototype for our new set of pilasters flanking the portals (Figure 20).

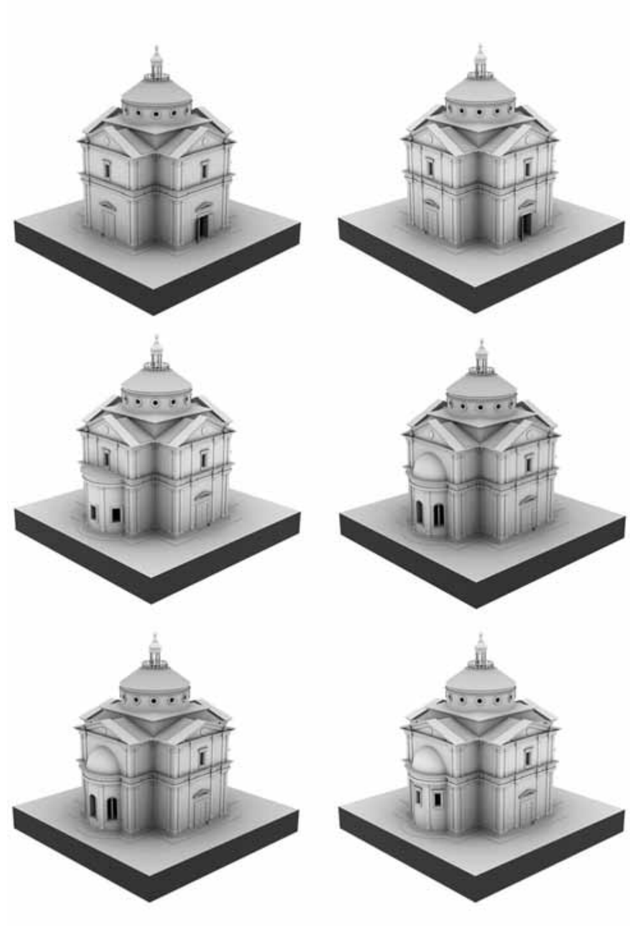

Figure 19 Santa Maria delle Carceri, Prato, development of the semicircular apse solution for the northern façade (derived from *Taccuino senese,* fol. 19r, ms. S. IV. 8, Biblioteca Comunale degli Intronati, Siena; images by the authors).

Other changes included modification of the interior decorations above the main portals in accordance with the new dimensions and extension of the existing white-and-blue decorative frieze on the entablature by Andrea della Robbia into the new circular apse (Figure 21). All later additions were removed (the balustrade in front of the altar, the two aediculae adjacent to the nave, and floor finishes). The classicizing semicircular niches shown in the *Taccuino* plan correspond to those on the interior walls, with a governing radius of 1 *braccio fiorentino*. The head of each semicircular niche rests on a stringcourse that runs behind the main order of the interior walls, following the design of the Cappella Pazzi in Florence. This is perhaps one of our most approximate solutions.[41] The vertical proportions of the niches conform to the proportions of the church's upper windows, their framing corresponds to the framing of the entrance portal, and their heights were documented by the survey.

In terms of procedure, the parametric approach ensures that the proportions of the model correspond to Sangallo's

Figure 20 Santa Maria delle Carceri, Prato, pilasters flanking entrance portals (derived from *Taccuino senese,* fol. 19r, ms. S. IV. 8, Biblioteca Comunale degli Intronati, Siena; image by the authors).

design logic, as revealed by both the drawing in the *Taccuino senese* and the existing building. At the same time, this approach provides new evidence that allows us to update previous solutions, and it also facilitates continuous editing and adaptation of new updates. We assembled these digital objects using Autodesk Revit, a BIM software technology that expresses the intrinsic parametric nature of architectural components.

As noted, Sangallo's measurements provided us with dimensional rules that then guided the behavior of our parameters: *braccia fiorentine*, *soldi*, and *denari* offered numerical constraints that could be adjusted until their proportions, driven by parametric geometry, replicated those of the actual decorations and spaces. The successful application of this technology demonstrates its value for these reconstructive investigations.

We extracted elements of Sangallo's architectural grammar from details sketched in the *Taccuino*, generating a library of parametric elements to support the final digital assembly. Given the many possible variables that could have affected the design for the church elevations, we tested several hypotheses before making a final selection of the most plausible elements. Again, architectural proportions drove the decisions governing our digital model, as evident, for example, in interior elevations featuring semicircular niches (see Figure 21).

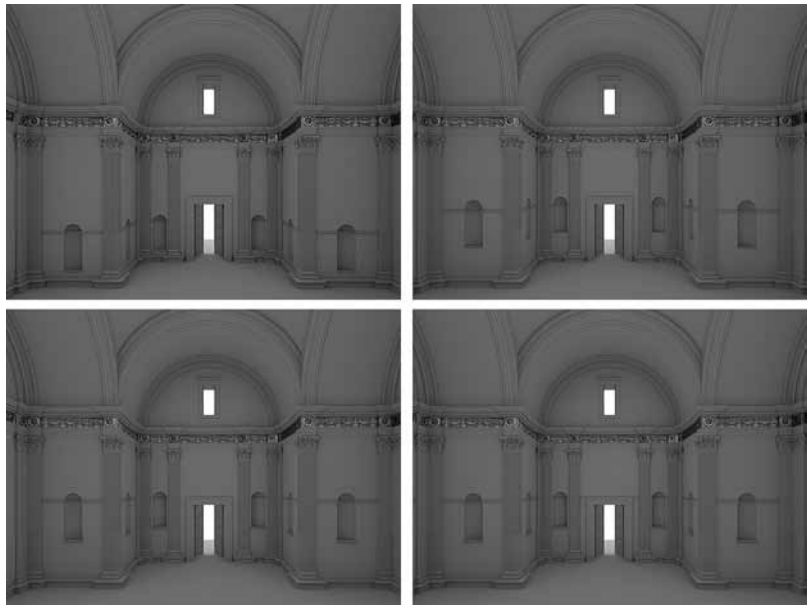

Figure 21 Santa Maria delle Carceri, Prato, development of semicircular niche arrangements for interior façades (derived from *Taccuino senese,* fol. 19r, ms. S. IV. 8, Biblioteca Comunale degli Intronati, Siena; images by the authors).

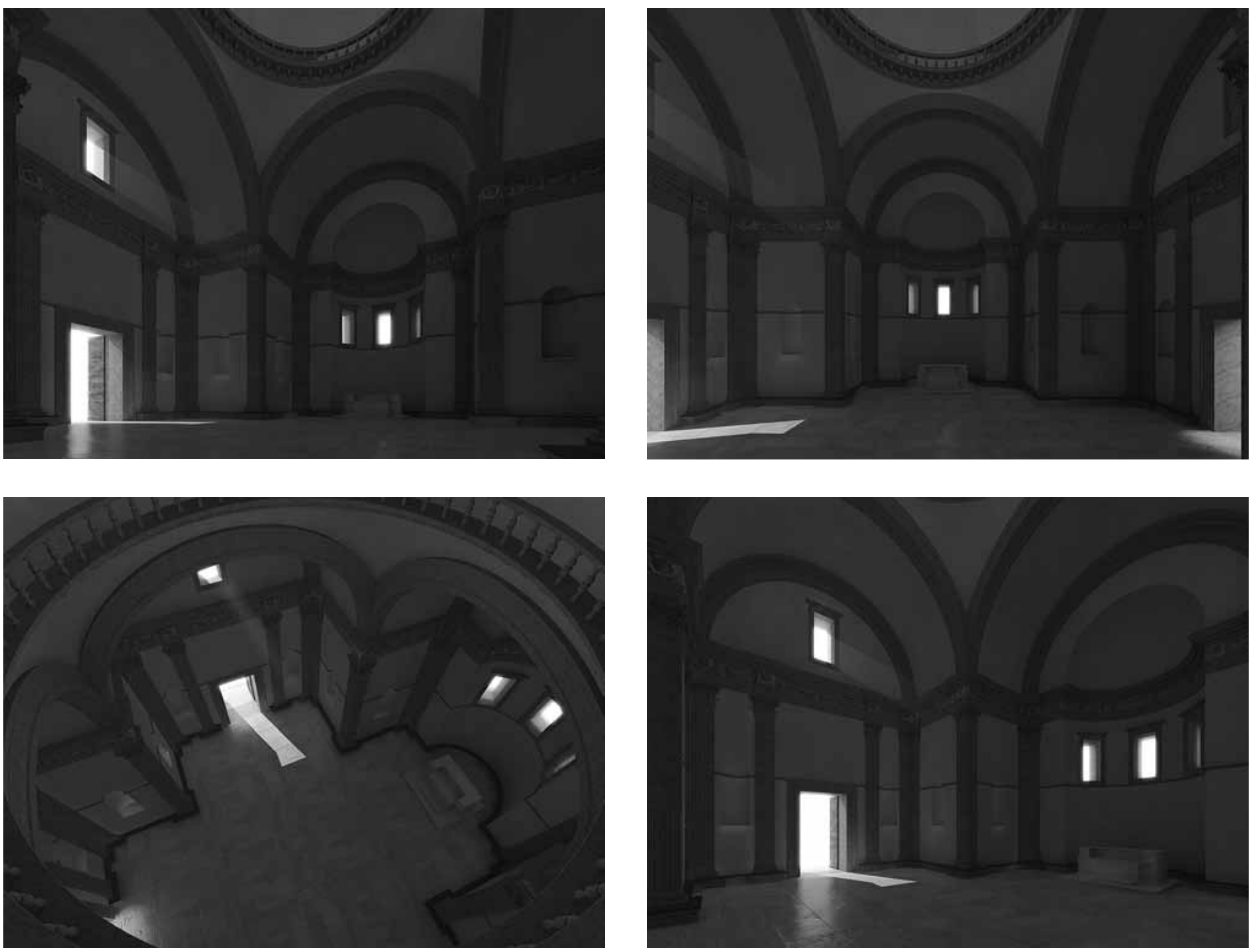

Figure 22 Santa Maria delle Carceri, Prato, photorealistic renderings of the reconstructed interior (images by the authors).


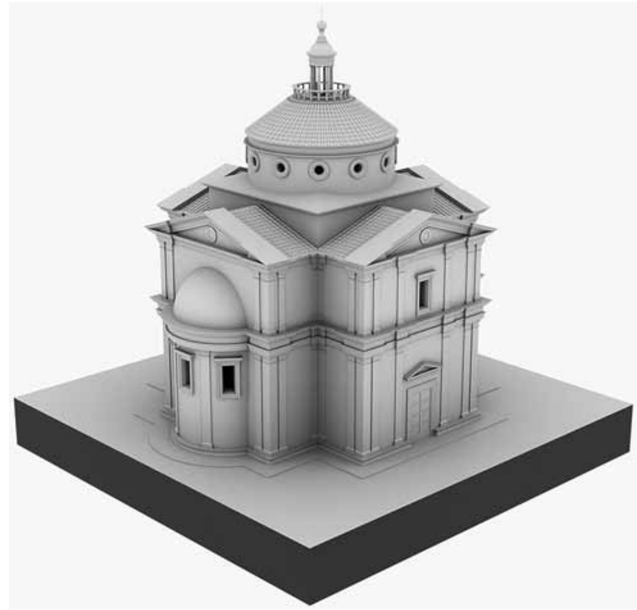

Figure 23 Santa Maria delle Carceri, Prato, photorealistic renderings of the reconstructed exterior (images by the authors).

The use of BIM technology made it possible for us to test different hypotheses about the original design of Santa Maria delle Carceri as well as to gain new knowledge about Sangallo's design process. Our synthetic global reconstruction integrated the proportions used by Sangallo in his drawings and writings with evidence from the built form of the church itself, generating several models that shed new light on the connections between the *Taccuino senese* plan and the existing structure. The complexity of this building analysis is even more impressive when we consider the extraordinary level of detail required to reproduce the elaborate moldings, pilasters, capitals, and frames ornamenting both the interior and the exterior of Sangallo's architecture (Figures 22 and 23). Through the creation of these 3-D semantic data models, we gain not only more precise knowledge of Giuliano da Sangallo's architectural work but also a better understanding of the close connection between theory and practice in the Sangallo workshop.

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Notes

1. In the digital field, the term *semantics* refers to the meanings of concepts, properties, and relationships that formally represent real-world entities, events, and scenes in a logical underpinning, such as a description logic. The meanings of description logic concepts and roles are defined by their model-theoretic semantics, which are based on interpretations. Building a digital semantic 3-D model entails the use of a general concept in software engineering for which a conceptual data model includes semantic information. This means that the model describes the meanings of its instances. Such a semantic data model is an abstraction that defines how the stored symbols (the instance data) relate to the real world. Specifically, in architecture, digital semantic 3-D models encode not only the geometry (the usual requirement for 3-D model construction) but also the identities of a building's key components, such as walls, floors, and ceilings, with their locations and geometric shapes. Digital semantic 3-D models embed a series of structured objects using a specific architectural lexicon, with the final goal of creating representations not simply for visualization purposes but for the clarification of architectural information, such as a building's shape (spatial extension), its structures (entity collections and part–whole relationships), its quality attributes (colors, textures, and the like), and its interaction with its time period. Digital models have multiple uses for historical analysis and reconstruction related to a building's current state, its morphology interpretation, and its transformations over time. On this topic, see Mathieu Koehl and Pierre Grussenmeyer, "Semantic Aspects for 3D Modeling," *International Archives of Photogrammetry* *and Remote Sensing* 33, pt. B4 (2000), 507–14; Livio De Luca, "3D Modelling and Semantic Enrichment in Cultural Heritage," in *Photogrammetric Week '13*, ed. Dieter Fritsch (Berlin: Wichmann/VDE Verlag, 2013), 323–34. For specific discussion of digital semantic 3-D models as applied to Giuliano da Sangallo's architecture, see Sabine Frommel, Marco Gaiani, and Simone Garagnani, "Progettare e costruire durante il Rinascimento: Un metodo per lo studio di Giuliano da Sangallo," *Disegnare idee immagini* 56 (2018), 20–31.

2. Caroline Bruzelius, "Digital Technologies and New Evidence in Architectural History," *JSAH* 76, no. 4 (Dec. 2017), 436–39.

3. Sabine Frommel, "Les maquettes de Giuliano da Sangallo," in *Les maquettes d'architecture: Fonction et évolution d'un instrument de conception et de réalisation*, ed. Sabine Frommel (Rome: Campisano Editore, 2015), 75–86.

4. Giuliano da Sangallo, "Progetto in pianta per la chiesa di Santa Maria delle Carceri," *Taccuino senese*, fol. 19r, ms. S. IV. 8, Biblioteca Comunale degli Intronati, Siena. The *Taccuino senese* comprises fifty-two small sheets of parchment (each 179–81 by 119–25 millimeters) with representations of architecture, antiquities, mechanics, and ornamentation carefully made in brown ink, often with preparatory drawings. The folios extend over a wide chronological field, ranging from the eighties of the quattrocento to the end of the second decade of the sixteenth century; however, the collection is mostly representative of the years of Sangallo's Florentine phase and slightly beyond, from 1480 to 1500, before his Roman period. It is particularly distinguished from the architect's other collection, the *Codex Barberini*, by the large number of representations of projects made in the late fifteenth century, including the church of Santa Maria delle Carceri in Prato. See Sabine Frommel, "Le architetture del *Taccuino senese*: La Sapienza di Siena, la villa di Poggio a Caiano, Santa Maria delle Carceri a Prato, disegni per Santo Spirito e San Lorenzo," in *Giuliano da Sangallo: Disegni degli Uffizi*, ed. Dario Donetti, Marzia Faietti, and Sabine Frommel (Florence: Giunti Editore, 2017), 32–49. See also Chloé Demonet, "Théorie et pratique du relevé d'architecture au XV[e] siècle et au début du XVII[e]: Giuliano da Sangallo et ses contemporains" (PhD thesis, École Pratique des Hautes Études, Paris Sciences & Lettres, forthcoming).

5. The term *parametric* originates from mathematics (parametric equation) and refers to the use of certain parameters or variables that can be edited to manipulate or alter the result of an equation or system. Parametric modeling is based on algorithmic thinking that enables a series of preprogrammed rules or algorithms (known as parameters) that together define, encode, and clarify the relationship between design intent and design response. That is, the model (or elements of it) is generated automatically by internal logic arguments rather than through manipulation. Typically, parametric rules create relationships between different elements of the design. A major example of parametric design is classical order, where, from a known column half diameter, one can reconstruct all other elements using simple algorithms based in proportions. The same algorithm used to establish an object's shape might be used throughout a model, so that if an element or rule is changed, it changes throughout. In effect the model is a representation of all the rules that the user has defined.

6. Piero Morselli and Gino Corti, *La chiesa di Santa Maria delle Carceri in Prato: Contributo di Lorenzo de' Medici e Giuliano da Sangallo alla progettazione* (Florence: EDAM, 1982).

7. For examples of the rich scientific literature in this regard, see Giuseppe Marchini, *Giuliano da Sangallo* (Florence: Sansoni, 1942), 20–26, 87; Silvestro Bardazzi, Eugenio Castellani, and Francesco Gurrieri, *Santa Maria delle Carceri a Prato* (Florence: Grafiche Senatori, 1979); Stefano Borsi, *Giuliano da Sangallo: I disegni di architettura e dell'antico* (Rome: Officina Edizioni, 1985), 417–20; Riccardo Pacciani, "Santa Maria della Pietà a Bibbona e Santa Maria delle Carceri a Prato," in *La chiesa a pianta centrale: Tempio civico del Rinascimento*, ed. Bruno Adorni (Milan: Electa, 2002), 81–95; Paul Davies, "The Madonna delle Carceri in Prato and Italian Renaissance Pilgrimage Architecture," *Architectural History* 36 (1993), 1–18; Claudio Cerretelli, "Da oscura prigione a tempio di luce: La costruzione di Santa Maria delle Carceri a Prato," in *Santa Maria delle Carceri a Prato: Miracoli e devozione in un santuario toscano del Rinascimento*, ed. Anna Benvenuti (Florence: Mandragora, 2005), 45–95; Sabine Frommel, *Giuliano da Sangallo* (Florence: Edifir, 2014), 49–69; Sabine Frommel, "Le sanctuaire de Santa Maria delle Carceri à Prato: Un lieu dédié à la Vierge miraculeuse," in *Art Sacré* 32 (2016), 122–35; Paul Davies, "Giuliano da Sangallo e decorum negli edifici a pianta centrale: Santa Maria delle Carceri e la sagrestia di Santo Spirito," in *Giuliano da Sangallo*, ed. Amedeo Belluzzi, Caroline Elam, and Francesco Paolo Fiore (Milan: Officina Libraria, 2017), 304–18; David Hemsoll, "L'idea dell'architettura di Giuliano da Sangallo, 1485–1492," in Belluzzi et al., *Giuliano da Sangallo*, 121–32; Frommel, "Le architetture del *Taccuino senese*." For more biographical details, see Frommel, *Giuliano da Sangallo*, as well as the German edition, Sabine Frommel, Giuliano da Sangallo. *Architekt der Renaissance. Leben und Werk*, Birkhäuser, Basel, 2020); see also Belluzzi et al., *Giuliano da Sangallo*.

8. Jens Niebaum, *Der kirchliche Zentralbau der Renaissance in Italien: Studien zur Karriere eines Baugedankens in Quattro-und fruhen Cinquecento* (Munich: Hirmer, 2016), 1:184–205, 2:472–86.

9. Leonardo da Vinci, 1487–1490, Manuscript B, Bibliothèque de Institut de France, Paris, cited in Jean Guillaume, "Léonard et l'architecture," in *Léonard de Vinci: Ingénieur et architecte*, ed. Pierre Théberge (Montreal: Musée des Beaux-Arts de Montréal, 1987), 224–41.

10. On the Cathedral of Our Lady of Loreto, see Christoph L. Frommel, *L'architettura del santuario e del palazzo apostolico di Loreto da Paolo II a Paolo III* (Loreto: Edizioni Tecnostampa, 2018).

11. Richard Krautheimer, "Introduction to an 'Iconography of Medieval Architecture,' " *Journal of the Courtald and Warburg Institutes* 5 (1942), 1–33, reprinted in *Studies in Early Christian, Medieval, and Renaissance Art*, ed. James Sloss Ackerman et al. (New York: New York University Press, 1969).

12. See Niebaum, *Der kirchliche Zentralbau der Renaissance in Italien*, 1:185.

13. It is uncertain whether two ground plans for an octagonal building (1606A and 1607A, Gabinetto Disegni e Stampe degli Uffizi) have ties to this project. See the following note.

14. Paul Davies, "The Early History of S. Maria delle Carceri in Prato," *JSAH* 54, no. 3 (Sept. 1995), 329–33. See also Frommel, "Le architetture del *Taccuino senese*," 39–40, 48–49. The attribution of this design to Giuliano da Maiano has been contested by some scholars. However, after comparing the details of the drawings and the previous buildings, Niebaum concluded that the plans constitute Giuliano da Maiano's early projects for the sanctuary. Niebaum, *Der kirchliche Zentralbau der Renaissance in Italien*, 1:189. Nevertheless, the annotations are not in da Maiano's handwriting, meaning the drawings were made by someone in his *bottega* (shop) or by a copyist.

15. See Davies, "Early History of S. Maria delle Carceri," 329–33. The Dieci di Balìa was a temporary magistracy whose members were summoned when unforeseen events endangered Florence.

16. Thanks to Lorenzo's patronage, Alberti's treatise was published in 1485.

17. See Pacciani, "Santa Maria della Pietà a Bibbona."

18. Niebaum, *Der kirchliche Zentralbau der Renaissance in Italien*, 1:197.

19. In April 1485 the city council issued regulations intended to preserve the ancient prisons on the north side, where the miraculous painting had appeared following the apparition of the Virgin Mary.

20. On this page, the portico of the church's ground floor shows columns instead of pilasters. See Frommel, "Le architetture del *Taccuino senese*."

21. The virtual reconstructions presented here were developed as a result of a collaboration between the Renaissance Art History Department at the École Pratique des Hautes Études, Paris Sciences & Lettres, and the Department of Architecture at the University of Bologna, a collaboration founded on an ideal of cooperation between the disciplines of the history of architecture and design, and specializing in virtual models. A detailed report of the survey appears in the next section of this article.

22. On the fortress and prisons, see Cerretelli, "Da oscura prigione a tempio di luce," 50, figs. 48, 60, 61. On Sangallo's sources, see Davies, "Giuliano da Sangallo e decorum negli edifici a pianta centrale," 304–18; Frommel, *Giuliano da Sangallo*, 66.

23. Regarding the drawing in the *Taccuino senese*, see Cerretelli, "Da oscura prigione a tempio di luce"; Niebaum, *Der kirchliche Zentralbau der Renaissance in Italien*, 1:184–205, 2:472–86; Frommel, "Le architetture del *Taccuino senese*," 38. On the plan's proportions, see Morselli and Corti, *La chiesa di Santa Maria delle Carceri in Prato*, 38–53.

24. Davies, "Early History of S. Maria delle Carceri," 326–35.

25. The niches were initially previewed for the lateral walls of the arms and were then also added to both sides of the doorway. Niebaum, *Der kirchliche Zentralbau der Renaissance in Italien*, 1:198. It seems that some evidence of a niche was found during restoration work. See Marchini, *Giuliano da Sangallo*, 87.

26. Cerretelli, "Da oscura prigione a tempio di luce," 81.

27. The pilasters of the ground plan of the *Taccuino senese* clearly show such a correspondence, which remains obvious today in the section where the Doric order entablature and façade reach the level of the Corinthian order of the inner space.

28. Livio De Luca, "Methods, Formalisms and Tools for the Semantic-Based Surveying and Representation of Architectural Heritage," *Applied Geomatics* 6, no. 2 (June 2014), 115–39.

29. Building information modeling is a process supported by various tools and technologies involving the generation and management of digital representations of physical and functional characteristics of places. BIMs, or building information models, are files that can be extracted, exchanged, or shared to support decision making regarding a built asset. BIMs involve representing designs as combinations of "objects," usually consisting of solid-shape primitives. For discussion of recent digital innovations, see Fabrizio Ivan Apollonio, Marco Gaiani, and Zheng Sun, "3D Modeling and Data Enrichment in Digital Reconstruction of Architectural Heritage," *International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences* 40-5/W2 (2013), 43–48.

30. The folios of the *Codex Barberini* measure approximately 455 by 390 millimeters; those of the *Taccuino senese* are approximately 120 by 180 millimeters in size. Giuliano da Sangallo, *Codex Barberini*, ms. Barb. Lat. 4424, Biblioteca Apostolica Vaticana.

31. TLS is a ground-based, active imaging method that rapidly acquires accurate, dense 3-D point clouds of object surfaces through laser range finding.

32. Regarding measurement units, see Gloria Nobili, " 'Delle misure d'ogni genere antiche e moderne': Un inventario delle unità di misura premetriche," in *Atti del XVII Congresso Nazionale di Storia della Fisica e dell'Astronomia*, ed. Pasquale Tucci (Milan: Istituto di Fisica Generale Applicata, 1997).

33. These details were noted previously in Frommel, "Le architetture del *Taccuino senese*," 41.

34. 22.2 × 14.8 millimeters APS-C CMOS sensor, 3,888 × 2,592 pixels, with 35 millimeters of nominal focal length.

35. Morselli and Corti, *La chiesa di Santa Maria delle Carceri in Prato*, 55–186. A total station is an electronic/optical instrument used for surveying. It is an electronic transit theodolite integrated with electronic distance measurement (EDM) to measure both vertical and horizontal angles and the slope distance from the instrument to a particular point; it includes an onboard computer to collect data and perform triangulation calculations. A TS can be used to record absolute locations to integrate manual measurements.

36. Maurizio Filiaggi, in Morselli and Corti, 38–53.

37. For support for this argument, see Davies, "Giuliano da Sangallo e decorum negli edifici a pianta centrale."

38. Giuliano da Sangallo, order proportions, *Taccuino senese*, fol. 31v, ms. S. IV. 8, Biblioteca Comunale degli Intronati, Siena.

39. Marcus Vitruvius Pollio, *De architectura*, ed. Pierre Gros (Turin: Giulio Einaudi Editore, 1997), 4.5.6 (387–92).

40. It is possible that the pediment was created by Piero de' Medici between the death of Lorenzo and the exile of the dynasty in 1494, or even after the return of the family and the election of Leo X, Lorenzo's son, in 1513.

41. The Cappella Pazzi's niches are actually blind. The niches in the model follow a consistent shape that draws upon other buildings and projects from the same historical period. Digital semantic 3-D modeling will facilitate new research on this topic.